

A simple Method To Calculate Transverse Spin Structure

Function $g_2(x, Q^2)$

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Abstract

We calculated the spin dependent structure functions g_2^{ww} in the valon model representation of hadrons. A simple approach is given for the determination of the twist-3 part of $g_2(x, Q^2)$ structure function. Thus, enabling to obtain the full transverse structure function, $g_2(x, Q^2)$ of proton, neutron and the deuteron.

1 INTRODUCTION

The nucleon polarized structure functions $g_{1,2}(x, Q^2)$ are important tools in the investigation of the nucleon substructure. In particular, they are vital elements for the understanding of the spin dependent parton distributions and their correlations. The $g_2(x, Q^2)$

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structure function is important because it probes both transverse and longitudinally polarized parton distributions inside the nucleon. However, this function is also sensitive to the higher twist effects, such as quark gluon correlations which is not easily interpreted in pQCD, where such effects are not included and does not fade away even in big Q^2 . So it seems to be very interesting to calculate it, because it is the only function which is related to quark-gluon interaction. Properties of g_2 can be clearly established in DIS using the Operator Product Expansion(OPE). Therefore, it makes possible to study contributions to the nucleon spin structure beyond the simple quark parton model.

The main purpose of this paper is to calculate transverse spin structure function, $g_2(x, Q^2)$. It requires to consider the twist-2 and the twist-3 contributions. While the twist-2 part is well understood, we present a simple method to extract twist-3 part. Since twist-2 part of g_2 requires the knowledge about g_1 structure function, a brief review of the latter will also be given. In doing so, we utilize the so-called valon model representation of the hadrons. Finally we compare our results with the experimental data from E143 [1], E155 [2, 3], and also with newly released data from HERMES [4, 5] groups. A comparison of our results with other phenomenological models is also given.

The organization of the paper is as follows. In section 2, we briefly review how to calculate the polarized nucleon structure function in the valon model. In section 3 we calculate the $g_2(x, Q^2)$ spin structure function. We also discuss the numerical results, sum rules and the effect of higher twist on $g_2(x, Q^2)$. Then, we will finish with the conclusions.

2 A brief review of spin structure functions in the valon model

The valon model is a phenomenological model, originally proposed by R. C. Hwa, [6] in early 80's. It was improved later by Hwa [7] and Others [8, 9, 10, 11, 12, 13], and extended

to the polarized cases [14, 15, 16, 17]. In this model a hadron is viewed as three (two) constituent quark like objects, called valon. Each valon is defined to be a dressed valence quark with its own cloud of sea quarks and gluons. The dressing processes are described by QCD. The structure of a valon is resolved at high Q^2 . At low Q^2 , the internal structure of the valons can not be resolved and they behave as constituent quarks of the hadron. In this model the polarized parton distribution in a polarized hadron is given by:

$$\delta q_i^h(x, Q^2) = \sum \int_x^1 \frac{dy}{y} \delta G_{valon}^h(y) \delta q_i^{valon}(\frac{x}{y}, Q^2) \quad (1)$$

where $\delta G_{valon}^h(y)$ is the helicity distribution of the valon in the hosting hadron; in other words, it is the probability of finding a polarized valon inside the polarized hadron. The Q^2 dependence of $\delta G_{valon}^h(y)$ in the next to leading order is very marginal. Moreover, it is independent of the nature of the prob. The term $\delta q_i^{valon}(x/y, Q^2)$ in Eq. (1) is the polarized parton distribution inside a valon. These polarized parton distribution functions (PPDFs) are governed by the DGLAP evolution equations. The polarized hadron structure functions are obtained via a convolution integral as follows:

$$g_1^h(x, Q^2) = \sum_{valon} \int_x^1 \frac{dy}{y} \delta G_{valon}^h(y) g_1^{valon}(\frac{x}{y}, Q^2) \quad (2)$$

where $g_1^{valon}(\frac{x}{y}, Q^2)$ is the polarized structure function of the valon. The details of actual calculations are given in [14, 16]. The results obtained for $g_1(x, Q^2)$ from this model are in excellent agreement with all available experimental data. In Fig. 1 we only present a sample of the results, and show that how it compares with the more elaborated global fits and the other models. we will use those findings here in calculating $g_2(x, Q^2)$.

3 Transverse spin-dependent structure function $g_2(x, Q^2)$

Polarized Deep Inelastic Scattering(DIS) mediated by photon exchange probes two spin structure functions: $g_1(x, Q^2)$ and $g_2(x, Q^2)$. If the target is transversely polarized, the total cross section is a combination of these two structure functions. Transverse spin

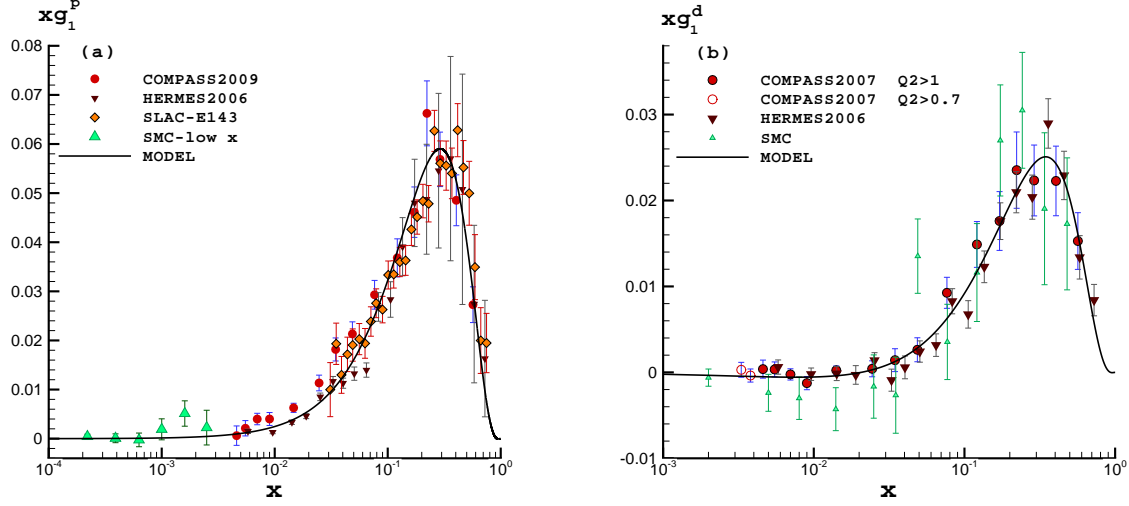


Figure 1: Polarized proton(deuteron) structure function at $Q^2 = 5(3)GeV^2$. The results from model [14] are compared with the experimental data [18, 19, 20, 21, 22, 23].

structure function, $g_2(x, Q^2)$, is made up of two components: a twist-2 part, g_2^{ww} , and a mixed twist part, $\bar{g}_2(x, Q^2)$. It can be written as [24]

$$g_2(x, Q^2) = g_2^{ww}(x, Q^2) + \bar{g}_2(x, Q^2) \quad (3)$$

where

$$\bar{g}_2(x, Q^2) = - \int_x^1 \frac{\partial}{\partial y} \left(\frac{m}{M} h_T(y, Q^2) + \xi(y, Q^2) \right) \frac{dy}{y}. \quad (4)$$

The twist-2 part, g_2^{ww} , comes from OPE. The $\bar{g}_2(x, Q^2)$ receives a contribution from the transversely polarized quark distributions $h_T(x, Q^2)$ as well as a twist-3 component. The twist-3 part, $\xi(y, Q^2)$ in Eq. (4) comes from quark-gluon interactions. These higher twist corrections arise from the non-perturbative multi parton interactions, whose contributions at low energy increase as $\frac{1}{Q^2}$, reflecting the confinement. The twist-3 term represents qqq correlations. Therefore, any non-zero result for this term at a given Q^2 will reflect a departure from the non-interacting partonic regime [25].

g_2^{ww} is related to the g_1 structure function by the Wandzura-Wilczek relation[26] as follows,

$$g_2^{ww}(x, Q^2) = -g_1(x, Q^2) + \int_0^1 g_1(x, Q^2) \frac{dy}{y}. \quad (5)$$

There are two important and well known sum rules regarding $g_1(x, Q^2)$ and $g_2(x, Q^2)$.

The first one is OPE sum rule:

$$\Gamma_1^n = \int_0^1 x^n g_1(x, Q^2) dx = \frac{a_n}{2}, n = 0, 2, 4, \dots \quad (6)$$

$$\Gamma_2^n = \int_0^1 x^n g_2(x, Q^2) dx = \frac{1}{2} \frac{n}{n+1} (d_n - a_n), n = 2, 4, \dots \quad (7)$$

The second one is Burkhardt-Cottingham sum rule [27],

$$\int_0^1 g_2(x, Q^2) dx = 0 \quad (8)$$

The Burkhardt-Cottingham sum rule for $g_2(x, Q^2)$ at large Q^2 was derived from virtual Compton scattering dispersion relations. This sum rule does not follow from the OPE since the $n = 0$ sum rule is not defined for $g_2(x, Q^2)$ in Eq. (7). Since $g_2 = g_2^{ww} - \bar{g}_2$, combining this with Eq. (5) results in a third sum rule (though not independent). So, we have in our disposal the following 3 sum rules

$$\int_0^1 g_2^{ww}(x, Q^2) dx = 0 \quad (9)$$

$$\int_0^1 x^2 g_2^{ww}(x, Q^2) dx = -\frac{1}{3} a_2 \quad (10)$$

$$\int_0^1 x^2 \bar{g}_2(x, Q^2) dx = \frac{1}{3} d_2 \quad (11)$$

where, a_2 and d_2 are the matrix elements of twist-2 and twist-3 operators.

Now we are set to calculate $g_2^{proton}(x, Q^2)$. It will be done in two parts: First we evaluate the twist-2 part, $g_2^{ww}(x, Q^2)$, and then we take up the twist-3 part, $\bar{g}_2(x, Q^2)$. Combining the results from the two steps results in the full $g_2(x, Q^2)$.

3.1 Calculation of the twist-2 term, $g_2^{ww}(x, Q^2)$

We begin with Eq. (5). Since $g_1(x, Q^2)$ is known in the valon model [9], we utilize those results without any additional free parameter and evaluate the twist-2 part of $g_2(x, Q^2)$; namely $g_2^{ww}(x, Q^2)$, according to the Eq. (5). The results are shown in Fig. 2 for proton

and the deuteron. We have included in Fig. 2 the results from other phenomenological models for the sake of comparison [28, 29]. The experimental data are from E143 and E155 Collaborations [1, 2, 3].

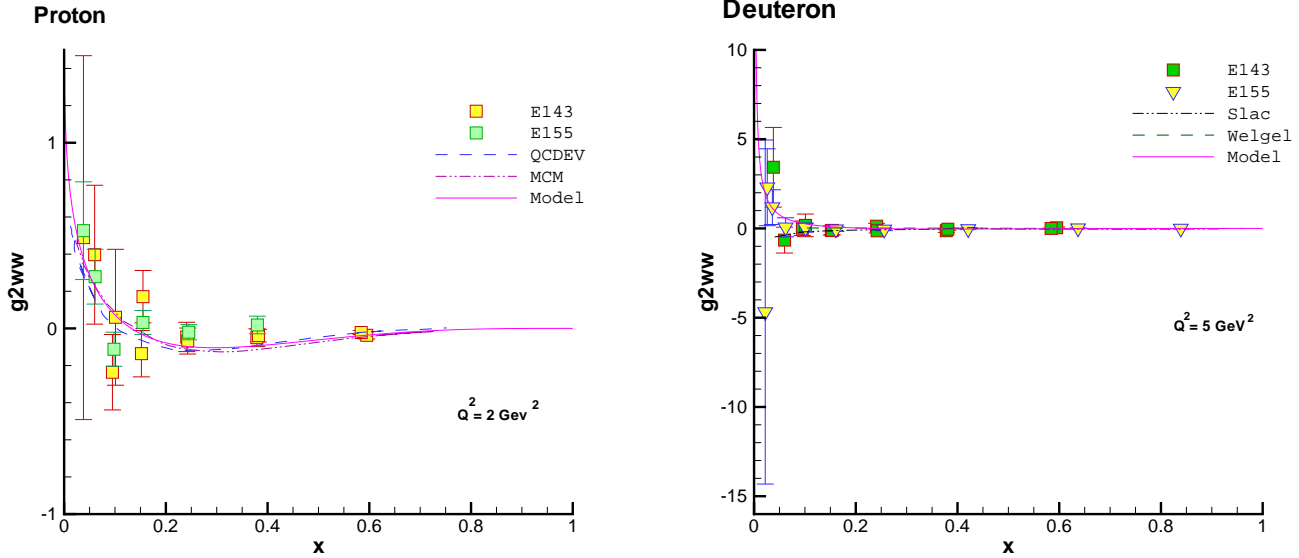


Figure 2: Transverse polarized proton and deuteron structure functions, $g_2^{ww}(x, Q^2)$ at $Q^2 = 2\text{GeV}^2$ and $Q^2 = 5\text{GeV}^2$.

3.2 Calculating the twist-3 term, $\bar{g}_2(x, Q^2)$

As mentioned before, the function $\bar{g}_2(x, Q^2)$ has two terms:

$$\bar{g}_2(x, Q^2) = - \int_x^1 \frac{\partial}{\partial y} \left(\frac{m}{M} h_T(y, Q^2) + \xi(y, Q^2) \right) \frac{dy}{y}. \quad (12)$$

The first term is related to the transverse polarization of quarks in the nucleon and is a twist-2 contribution. This term is suppressed by the quark to nucleon mass ratio, so we ignore this negligible part and focus on the second term which is the twist-3 contribution and related to quark-gluon correlations.

The moments of $\bar{g}_2(x, Q^2)$ obey the following simple equation [30]:

$$\bar{g}_2(n, Q^2) = L^{\frac{\gamma_n^g}{2b_0}} \bar{g}_2(n, Q_0^2) \quad (13)$$

Where,

$$\bar{g}_2(n, Q^2) = \int_0^1 x^{n-1} g_2(x, Q^2) dx, \quad (14)$$

$$L \equiv \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}, \quad (15)$$

$$b_0 = \frac{11}{3}N_c - \frac{2}{3}N_f \quad (16)$$

$$\gamma_n^g = 2N_c(S_{n-1} - \frac{1}{4} + \frac{1}{2n}) \quad (17)$$

$$S_{n-1} = \sum \frac{1}{j} \quad (18)$$

Our task is to solve Eq. (13) with some appropriate initial conditions in the moment space. Then we can make a transformation to the momentum space and evaluate the twist-3 contribution to the transverse spin structure function. This is done in two steps, as is the case in the valon model. The first step involves finding a solution to Eq. (13) in a valon. The second step is to convolute the results obtained in the first step with the valon distribution in the nucleon. This will give the nucleon structure function.

Our choice for the initial input function for $\bar{g}_2^{valon}(z, Q_0^2)$ is :

$$\bar{g}_2^{valon}(z, Q_0^2) = A\delta(z-1) \quad (19)$$

The justification for this choice is as follows: In the momentum space one can write

$$\bar{g}_2^{valon}(z, Q^2) = f(Q^2)\bar{g}_2(z, Q_0^2) = f(Q^2)A\delta(z-1) \quad (20)$$

Note that for $Q^2 = Q_0^2$ we get $f(Q^2) \rightarrow f(Q_0^2) = 1$ which is apparent from the definition of L in Eq. (15). Thus, we arrive at Eq. (19). This simple choice of the initial input for $\bar{g}_2^{valon}(z, Q_0^2)$ stems from the knowledge that it is related to the quark gluon correlations, which in turn, is related to the Green function in the momentum space. In the momentum space the correlation function is composed of a Dirac Delta term and a function that is related to the momentum. Therefore, at the initial Q_0^2 we can simply assume that $\bar{g}_2^{valon}(z, Q_0^2)$ is proportional to Dirac Delta function which emphasizes the conservation of energy- momentum and the fact that at that low Q_0^2 a valon behaves as an object without

any internal structure. The last point is built in the definition of a valon. So, in the moment space the Delta function becomes unity and we can write

$$g_2(n, Q_0^2) = A \times 1 \quad (21)$$

all the QCD effects are summarized in A . This coefficient will be extracted from the experimental data. With the above choice for the initial input, the moments of $\bar{g}_2^{valon}(z, Q^2)$ are obtained in the valon, using Eq. (12). An inverse Mellin transformation then takes us to the momentum space, giving $\bar{g}_2^{valon}(z, Q^2)$ structure function. This completes the first step described above. Meanwhile, a fit to E143 data [1] gives the following functional form for $\bar{g}_2^{valon}(z, Q^2)$ in the valon, depicted in Fig. 3.

$$\bar{g}_2^{valon}(z, Q^2 = 5GeV^2) = 0.0233z^{3.827}\delta(z-1) \quad (22)$$

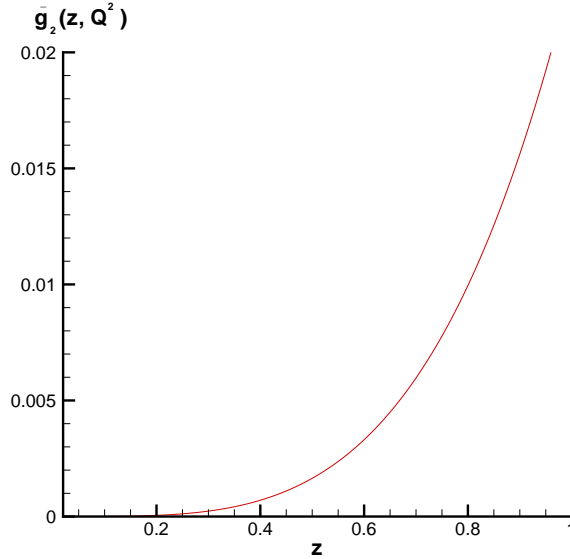


Figure 3: The twist-3 part of proton transverse spin structure function in the valon in z space, $\bar{g}_2(z, Q^2)$ at $Q^2 = 5GeV^2$.

In Fig. 4 we present $x^2\bar{g}_2^{p,d}$ for proton and deuteron along with the E143 data [1] data.

We have also compared our results with those from Bag model [28] and with the results obtained by Wakamatsu [31].

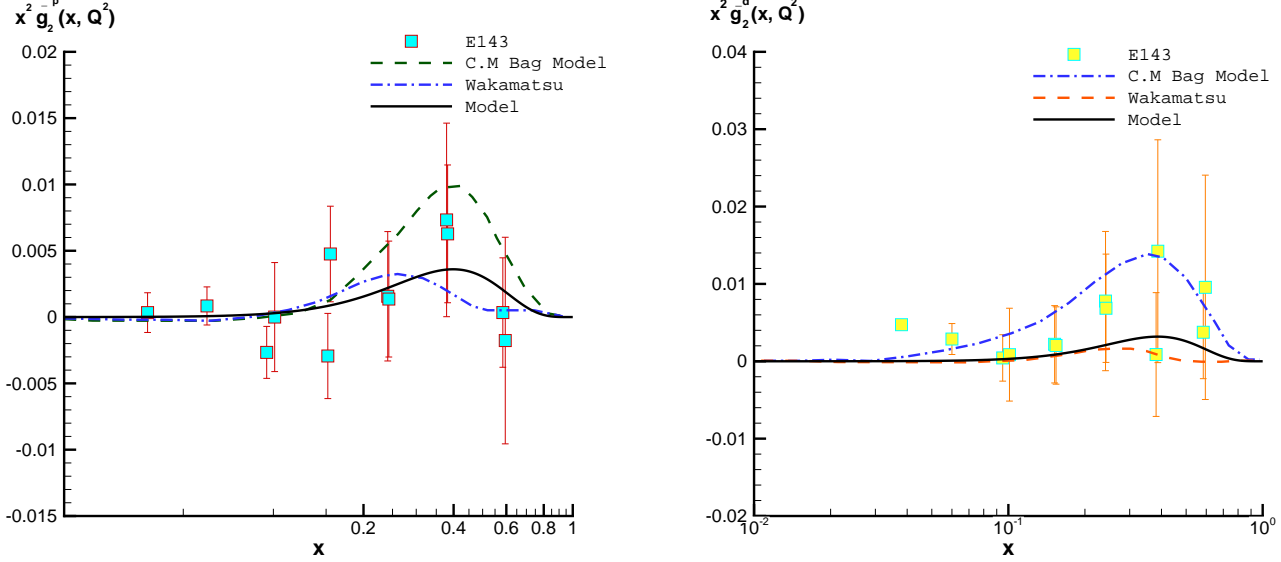


Figure 4: The twist-3 part of proton and deuteron transverse spin structure function, $x^2 \bar{g}_2(x, Q^2)$ at $Q^2 = 5 \text{ GeV}^2$.

The second step involves the convolution process which takes us to the hadronic level. This is similar to the way we have used to extract $g_1^{p,n}(x, Q^2)$, and described in a number of papers, e.g. see [14, 15, 16]. That is, briefly, to convolute $\bar{g}_2^{\text{valon}}(z, Q^2)$ with the valon helicity distribution $\delta G^{U(D)}(y)$ in the corresponding hadron to obtain $\bar{g}_2(x, Q^2)$. After adding $\bar{g}_2(x, Q^2)$ to the $g_2^{ww}(x, Q^2)$, the full g_2 is obtained and are shown in Fig. 5 for proton and deuteron and in Fig. 6 for helium-3 (For calculating the $g_2^{He3}(x, Q^2)$, we used the polarized proton and neutron helicity distributions which are derived in [32]).

Finally, in the table 1 we give the numerical values for the twist-2 and twist-3 matrix elements. They are defined in Eqs. (9) and (10), respectively. We also include results from other sources for the purpose of comparison. The results for the Burkhardt-Cottingham sum rule which is defined in Eq. (8) are given in table 2. This integral was evaluated

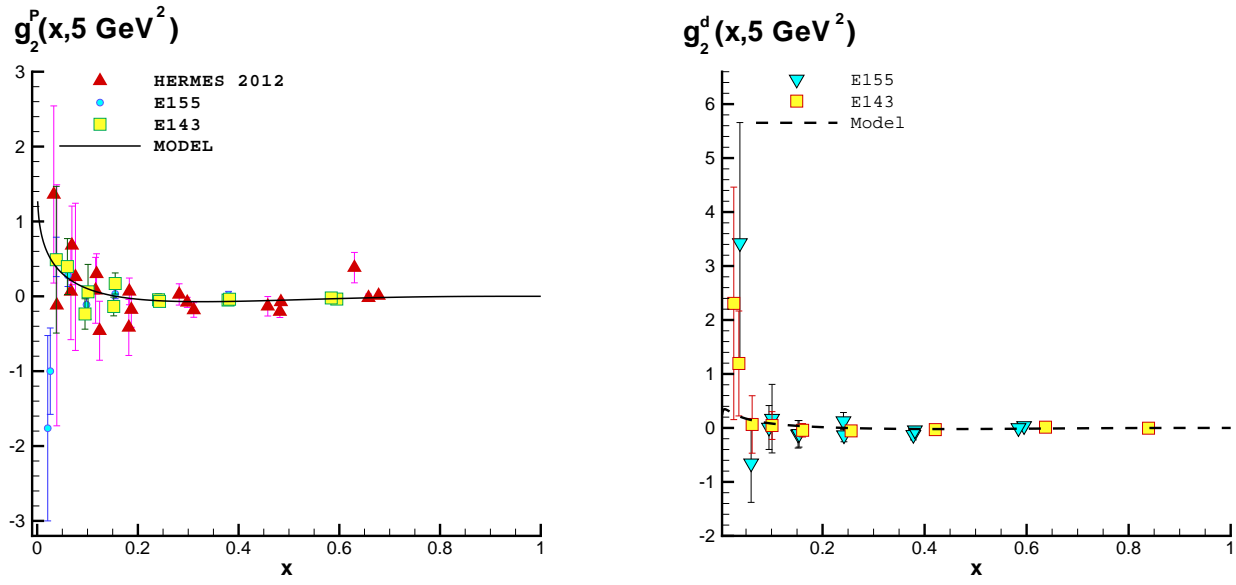


Figure 5: Full transverse polarized proton and deuteron structure function, $g_2(x, Q^2)$ at $Q^2 = 5 \text{ GeV}^2$.

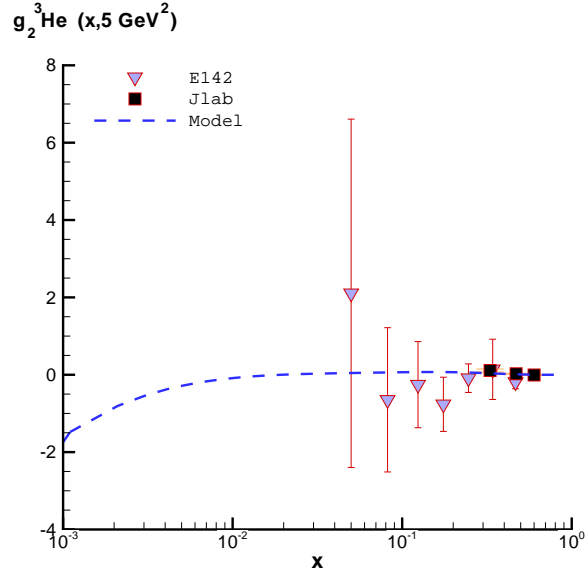


Figure 6: Full transverse polarized He^3 structure function, $g_2(x, Q^2)$ at $Q^2 = 5 \text{ GeV}^2$. We compared our results with experimental data from [33, 34].

	a_2^p	d_2^p	a_2^d	d_2^d
Valon model	0.0224	0.0042	0.010	0.0037
MIT bag model [28, 35]	—	0.01	—	0.005
QCD sum rule [36]	—	$-(0.6 \pm 0.3)10^{-2}$	—	-0.017
QCD sum rule [37]	—	$-(0.3 \pm 0.3)10^{-2}$	—	-0.013
Lattice QCD [38]	$(3 \pm 0.64)10^{-2}$	$-(4.8 \pm 0.5)10^{-2}$	$(13.8 \pm 5.2)10^{-3}$	-0.022
CM bag model by Song [28]	0.0210	0.0174	0.0087	0.0067
E143 [1]	$(2.42 \pm 0.20)10^{-2}$	$(0.54 \pm 0.5)10^{-2}$	$(8.0 \pm 0.16)10^{-3}$	$(3.9 \pm 9.2)10^{-3}$

Table 1: The twist-2 and twist-3 matrix element operators a_2 and d_2 , for the proton and the deuteron, calculated in the valon model. Also included the experimental data and the results from other theoretical investigations.

	bag model by Song[28]	E143[1]	E155[3]	HERMES 2012[5]	Valon model
$\int g_2^p(x, Q^2)dx$	-0.0016	-0.014 ± 0.028	-0.022 ± 0.071	$0.006 \pm 0.024 \pm 0.017$	-0.004
$\int g_2^d(x, Q^2)dx$	-0.00287	-0.034 ± 0.082	0.023 ± 0.044	-	0.010

Table 2: The results for the Burkhardt-Cottingham sum rule.

in the region of $0.023 < x < 0.9$ at $Q^2 = 5\text{GeV}^2$. We compared our results with those experimental data from HERMES in the same region and also with the results from E143 and E155 in the region of $0.02 < x < 0.8$. We compared our results with those from bag model too.

4 Conclusion

We have used the valon model and calculated the transverse spin structure function for nucleon and deuteron. We offer a simple approach for calculating the twist-3 part of the transverse spin structure function. It is evident that our results for both the twist-2 part and for the full transverse spin structure functions are in good agreements with the experimental data. We have also provided a comparison of our results with other works.

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